Motion of System of Particles and Rigid Body Important Questions With Answers

NEET Physics 2023

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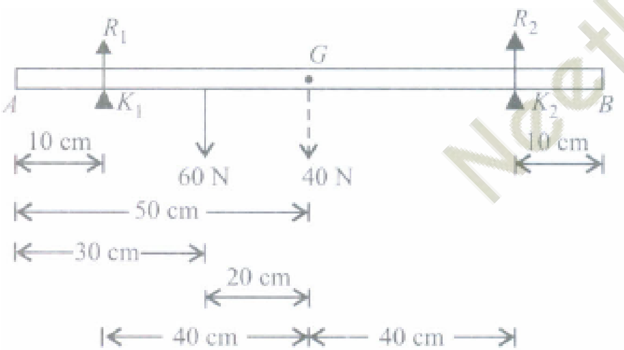
1. The moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its midpoint and perpendicular to its length is I_0, Its moment of inertia about an axis passing through one of its ends and perpendicular to its length is :
   a) $I_0 + (ML^2)$ b) $I_0 + (ML^2/2)$ **c) $I_0 + (ML^2/4)$** d) $I_0 + (2ML^2)$
   Solution : -
   By the theorem of parallel axes, the moment of inertia of a body about any axis is equal to the sum of the moment of inertia about a parallel axis through the centre of mass and Ma^2, where M is the mass of the body and a is the separation between the two axes.
   Therefore, the moment of inertia of the rod about an axis passing through one of its ends and perpendicular to its length is
   $I = I_0 + M(L/2)^2$
   $= I_0 + ML^2/4$
2. A uniform rod of length 1 m and mass 4 kg is supported on two knife-edges placed 10 cm from each end. A 60 N weight is suspended at 30 em from one end. The reactions at the knife edges is :
   a) 60 N, 40 N b) 75 N, 25 N **c) 65 N, 35 N** d) 55 N, 45 N
   Solution : -
   

   AB is the rod. K_1 and K_2 are the two knife edges.
   Since the rod is uniform, therefore its weight acts at its centre of gravity G.
   Let R1 and R2 be reactions at the knife edges.
   For the translational equilibrium of the rod,
   $R_1 + R_2 - 60\ N - 40\ N = 0$
   $R_1 + R_2 = 60\ N + 40\ N = 100\ N$ ... (i)
   For the rotational equilibrium, taking moments about G, we get
   $-R_1(40) + 60(20) + R_2(40) = 0$
   $R_1 - R_2 = \frac{1200}{40} = 30N$
   Adding (i) and (ii), we get $2R_1 = 130$ N or $R_1 = 65$ N Substituting this value in Eq. (i), we get $R_2 = 35$ N

3. Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-3\hat{i} - 2\hat{j} + \hat{k}$ respectively. The centre of mass of this system has a position vector________

a) $-2\hat{i} - \hat{j} + \hat{k}$ b) $2\hat{i} - \hat{j} - 2\hat{k}$ c) $-\hat{i} + \hat{j} + \hat{k}$ d) $-2\hat{i} + 2\hat{k}$

Solution : -

The position vector of the centre of mass of two particle system is given by

$\vec{R} = \frac{m_1\vec{R}_1 + m_2\vec{R}_2}{(m_1+m_2)}$

$= \frac{1}{4}[-8\hat{i} - 4\hat{j} + 4\hat{k}] = -2\hat{i} - \hat{j} + \hat{k}$

4. The force $7\hat{i} + 3\hat{j} - 5\hat{k}$ acts on a particle whose position vector is $\hat{i} - \hat{j} + \hat{k}$. What is the torque of a given force about the origin?

a) $2\hat{i} + 12\hat{j} + 10\hat{k}$ b) $2\hat{i} + 10\hat{j} + 12\hat{k}$ c) $2\hat{i} + 10\hat{j} + 10\hat{k}$ d) $10\hat{i} + 2\hat{j} + \hat{k}$

Solution : -

$Here, \vec{r} = \hat{i} - \hat{j} + \hat{k}, \vec{F} = 7\hat{i} + 3\hat{j} - 5\hat{k}$

Torque, $\vec{\tau} = \vec{r} \times \vec{F}$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix} = \hat{i}(5-3) + \hat{j}(7-(-5)) + \hat{k}(3-(-7))$$

or $\vec{\tau} = 2\hat{i} + 12\hat{j} + 10\hat{k}$

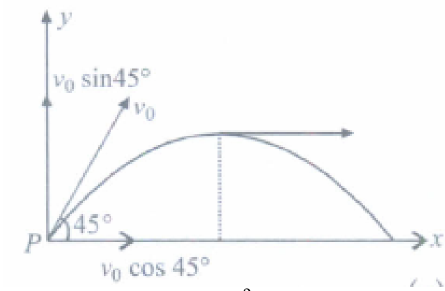
5. A particle is projected at time t = 0 from a point P on the ground with a speed v_o, at an angle of 45° to the horizontal The angular momentum of the particle about P at time t = v_o/g is

a) $\frac{mv_0^3}{2\sqrt{2}g}$ b) $\frac{mv_0^3}{\sqrt{2}g}$ c) $\frac{3mv_0^3}{\sqrt{2}g}$ d) $\frac{\sqrt{2}mv_0^3}{g}$

Solution : -

Let L = angular momentum of the particle
about P at time t = vo/g
At time t = vo/g:
Let P_x = x-component of momentum of particle
v_x = x-component of velocity of particle
x = displacement along x-axis.
Let P_y , V_y and y denote the quantities along y-axis
Angular momentum, L = x P_y - y P_x
or L = x (mv_y) - y(mv_x) or L = m[x v_y - yv_x] ....(i)
To find V_x and V_x:
V_x = Horizontal velocity of the projectile
or $v_x = v_0 cos45^0 = \frac{v_0}{\sqrt{2}}$ ...(ii)
It remains constant all along.
V_y = vertical velocity of particle
or $v_y = (v_0 sin45^0) - g \times \left(\frac{v_0}{g}\right) \quad [\therefore v = u + at]$
or $v_y = \left(\frac{v_0}{\sqrt{2}} - v_0\right) \quad \ldots.(iii)$
To find x and y:
x = V_x *X* time

or $x = \frac{v_0}{\sqrt{2}} \times \frac{v_0}{g} = \frac{v_0^2}{\sqrt{2}g} \therefore x = \frac{v_0^2}{\sqrt{2}g}$ ....(iv)



$y = v_0 sin\theta t - \frac{gt^2}{2}$

$y = \frac{v_0}{\sqrt{2}} \times \frac{v_0}{g} - \frac{g}{2}\left(\frac{v_0}{g}\right)^2$

or $y = \left(\frac{v_0^2}{\sqrt{2}g} - \frac{v_0^2}{2g}\right)$ ...(v)

$\therefore$ Substitute (ii), (iii), (iv) and (v) in (i)

$L = m\left[\frac{v_0^2}{\sqrt{2}g} \times \left(\frac{v_0}{\sqrt{2}} - v_0\right) - \left(\frac{v_0^2}{\sqrt{2}g} - \frac{v_0^2}{2g}\right)\frac{v_0}{\sqrt{2}}\right]$

or $L = \frac{mv_0^3}{g}\left[\left(\frac{1}{2} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2} - \frac{1}{2\sqrt{2}}\right)\right]$

or $L = \frac{mv_0^3}{g} \times \left(\frac{-1}{2\sqrt{2}}\right) = \frac{-mv_0^3}{2\sqrt{2}g}$

L is $\perp$ to plane of motion and is directed away from the reader i.e., along -ve z direction.

6. Two particles which are initially at rest, move towards each other under the action of their internal attraction. If their speeds are v and 2 v at any instant, then the speed of centre of mass of the system will be ____________ .
   a) 2 v **b) zero** c) 1.5 d) v

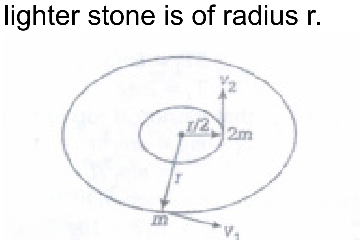
   Solution : -
   The centre of mass remains at rest, when no external force acts on a system of particles. So, speed of centre of mass is zero.

7. Which of the following is the correct relation between linear velocity $\vec{v}$ and angular velocity $\vec{w}$ of a particle?
   a) $\vec{v} = \vec{r} \times \vec{w}$ **b)** $\vec{v} = \vec{w} \times \vec{r}$ c) $\vec{w} = \vec{r} \times \vec{v}$ d) $\vec{w} = \vec{v} \times \vec{r}$

   Solution : -
   The relation between linear velocity v and angular velocity $\vec{w}$ is $\vec{v} = \vec{w} \times \vec{r}$

8. Two stones of masses m and 2m are whirled in horizontal circles, the heavier one in a radius r/2 and the lighter one in radius r. The tangential speed of lighter stone is n times that of the value of heavier stone when they experience same centripetal forces. The value of n is :
   a) 1 **b) 2** c) 3 d) 4

   Solution : -
   The two stones of masses m and 2m are whirled in horizontal circles where heavier stone is of radius r/2 and lighter stone is of radius r.



It is noted that as lighter stone is n times that of value of heavier stone, so when similar centripetal forces experience, then

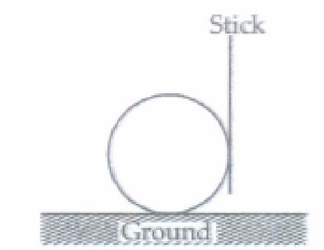
$(F_c)_{heavier} = (F_c)_{lighter}$

$2m(v)^2/(r/2) = m(nv)^2/r$

Now $n^2 = 4$

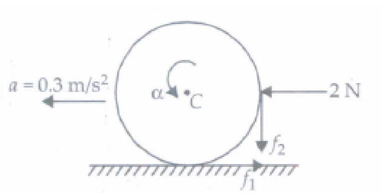
n = 2

9. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs. Then the coefficient of friction between the stick and the ring is :



a) 0.4 b) 0.8 c) 0.2 d) 0.5

Solution : -



As 2 - f_1 = Ma

f_1 = 2 - Ma = 2 - 2 x 0.3 = 1.4 N

Taking torque about C

$f_1 R - f_2 R = I_C \alpha$

$$(f_1 - f_2)R = MR^2 \frac{a}{R} \qquad (\therefore \alpha = \frac{a}{R})$$

$f_1 - f_2$ = Ma

$f_2 = f_1$ - Ma = 1.4 - 2 x 0.3 = 0.8 N

$f_2 = 2\mu$

$$\mu = \frac{0.8}{2} = 0.4$$

10. A solid cylinder of mass 20 kg and radius 20 cm rotates about its axis with an angular speed of 100 rad s^{-1}. The angular momentum of the cylinder about its axis is

a) 40 J s b) 400 J s c) 20 J s d) 200 J s

Solution : -

Here, M = 20 kg

R = 20 cm = 20 x 10^{-2} m, W= 100 rad s^{-1}

Moment of inertia of the solid cylinder about its axis is

$$I = \frac{MR^2}{2} = \frac{(20Kg)(20\times10^{-2}m)^2}{2} = 0.4kg \quad m^2$$

Angular momentum of the cylinder about its axis is

L = Iw = (0.4 kg m^2) (100 rad s^{-1}) = 40 J s

11. Two racing cars of masses m and 4 m are moving in circles of radii r and 2 r respectively. If their speeds are such that each makes a complete circle in the same time, then the ratio of the angular speeds of the first to the second car is____

a) 4: 1 b) 2: 1 **c) 1: 1** d) 8: 1

Solution : -

As both cars take the same time to'complete the circle and as $\omega=\frac{2\pi}{t}$, therefore ratio of angular speeds of the cars will be 1: 1.

12. To maintain a rotor at a uniform angular speed of 100 rad s^{-1}, an engine needs to transmit torque of 100 N m. The power of the engine is

a) 10 kW b) 100 kW c) 10 MW d) 100 MW

Solution : -

Here, w = 100 rad s^{-1}, τ = 100 N m

As P = τw

$\therefore$ P= (100 Nm) (l00 rad s^{-1}) = 10 X 10^3 W = 10 kW

13. A spherical ball rolls on a table without slipping. Then, the fraction of its total energy associated with rotation is_____

a) $\frac{2}{5}$ **b)** $\frac{2}{7}$ c) $\frac{3}{5}$ d) $\frac{3}{7}$

Solution : -

Concept The total kinetic energy of the ball rolling on a table without slipping is equal to its rotational kinetic energy and translational kinetic energy.

Total kinetic energy of spherical ball is given by K = Kinetic energy rotational $\left(K_{\text{rev}}\right)+$ Kinetic energy translational (K_{erans})

$=\frac{1}{2}I\omega^2+\frac{1}{2}mv^2$

For sphere, moment of inertia about its diameter

$I=\frac{2}{5}mr^2$

$\therefore K=\frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2+\frac{1}{2}mv^2$

$=\frac{1}{5}mr^2\omega^2+\frac{1}{2}mv^2$

$=\frac{1}{5}mv^2+\frac{1}{2}mv^2(\text{ as } v=r\omega)$

$=\frac{7}{10}mv^2$

$\therefore \frac{K_r}{K}=\frac{\frac{1}{5}mv^2}{\frac{7}{10}mv^2}=\frac{2}{7}$

14. A wheel has angular acceleration of $3.0\mathrm{rad}/\mathrm{sec}^2$ and an initial angular speed of $2.00\mathrm{rad}/\mathrm{sec}$. In a time of $2\mathrm{sec}$ it has rotated through an angle (in radian) of _____

a) 10 b) 4 c) 12 d) 6

Solution : -

Since, $\theta=\omega_0t+\frac{1}{2}\alpha t^2$

Where a is angular acceleration, ω_0 is the initial angular speed.

t =2 s

$\theta=2\times2+\frac{1}{2}\times3(2)^2=4+6=10\mathrm{rad}$

15. Assertion: If there are no external forces, the centre of mass of a double star moves like a free particle.

Reason: If we go to the centre of mass frame, then we find that the two stars are moving in a circle about the centre of mass, which is at rest.

a) If both assertion and reason are true and reason is the correct explanation of assertion.

b) If both assertion and reason are true but reason is not the correct explanation of assertion.

c) If assertion is true but reason is false. d) If both assertion and reason are false

Solution : -

In our frame of reference, the trajectories of the stars are a combination of

(i) uniform motion in a straight line of the centre of mass and

(ii) circular orbits of the stars about the centre of mass.

16. A disc is rotating with angular velocity & about its axis. A force $\vec{F}$ acts at a point whose position vector with respect to the axis of rotation is $\vec{r}$. The power associated with the torque due to the force is given by

a) $(\vec{r} \times \vec{F}).\vec{w}$ b) $(\vec{r} \times \vec{F}) \times \vec{w}$ c) $\vec{r}.(\vec{F} \times \vec{w})$ d) $\vec{r} \times (\vec{F}.\vec{w})$

Solution : -

Torque, $\vec{\tau} = \vec{r} \times \vec{F}$

$\therefore$ Power associated with the torque is

$p = \vec{\tau}.\vec{w} = (\vec{r} \times \vec{F}).\vec{w}$

17. A ballet dancer, dancing on a smooth floor is spinning about a vertical axis with her arms folded with an angular velocity of 20 rad/s. When she stretches her arms fully, the spinning speed decrease as 10 rad/s, If I is the initial moment of inertia of the dancer, the new moment of inertia is

a) 2*I* b) 3*I* c) *I/2* d) *I/3*

Solution : -

Here, angular momentum is conserved.

Initial angular momentum = Final angular momentum

I x 20 = I' x 10, where I' is new moment of inertia

$\therefore$ I' = 2*I*

18. Two discs are rotating about their axes, normal to the discs and passing through the centres of the discs. Disc D_1 has 2kg mass and 0.2 m radius and initial angular velocity of $50 \mathrm{rads}^{-1}$. Disc D_2 has 4 kg mass, 0.1 m radius and initial angular velocity of $200 \mathrm{rads}^{-1}$. The two discs are brought in contact face to face, with their axes of rotation coincident. The final angular velocity (in $\mathrm{rad}s^{-1}$) of the system is____

a) 40 b) 60 **c) 100** d) 120

Solution : -

We have,

$m_1 = 2\ \mathrm{kg} \quad m_2 = 4\ \mathrm{kg}$

$r_1 = 0.2\ \mathrm{m} \quad r_2 = 0.1\ \mathrm{m}$

$w_1 = 50 \mathrm{rads}^{-1} \quad w_2 = 200 \mathrm{rads}^{-1}$

$\text{As } I_1 W_1 = I_2 W_2 = \text{ Constant}$

$\therefore W_f = \frac{I_1 W_1 + I_2 W_2}{I_1 + I_2}$

$= \frac{\frac{1}{2} m_1 r_1^2 w_1 + \frac{1}{2} m_2 r_2^2 w_2}{\frac{1}{2} m_1 r^2 + \frac{1}{2} m_2 r_2^2}$

$= \frac{\frac{1}{2} \times 2 \times 0.04 \times 50 \frac{1}{2} + 4 \times 0.01 \times 200}{\frac{1}{2} \times 2 \times 0.04 + \frac{1}{2} \times 4 \times 0.01}$

$= 100 \mathrm{rads}^{-1}$

19. Assertion: A girl sits on a rolling chair, when she stretch her arms horizontally, her speed is reduced.

Reason : Principle of conservation of angular momentum is applicable in this situation.

a) If both assertion and reason are true and reason is the correct explanation of assertion

b) If both assertion and reason are true but reason is not the correct explanation of assertion

c) If assertion is true but reason is false d) If both assertion and reason are false.

Solution : -

If friction in the rotational mechanism is neglected, there is no external torque about the axis of rotation of the chair and hence Iω is constant, where 1 is moment of inertia and ω is angular velocity. Stretching the arms increases 1 about the rotation, results in decreasing the angular speed ω. Bringing the arms closer, body has the opposite effect.

20. Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities ω_1 and ω_2. They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is:

a) $\frac{1}{4}I(\omega_1-\omega_2)^2$ b) $I(\omega_1-\omega_2)^2$ c) $\frac{1}{8}I(\omega_1-\omega_2)^2$ d) $\frac{1}{2}I(\omega_1+\omega_2)^2$

Solution : -

According to the problem,

$I\omega_1 + I\omega_2 = 2I\omega_0$

$\omega_0 = (\omega_1 + \omega_2)/2$

$K_i = \frac{1}{2}I(\omega_1^2+\omega_2^2)$

$K_f = \frac{1}{4}(\omega_1+\omega_2)^2$

loss $\Delta K = I[\omega_1^2/2+\omega_2^2/2-\omega_1^2/4-\omega_2^2/4-2\omega_1\omega_2/4]$

$= I[\omega_1^2/4+\omega_2^2/4-2\omega_1\omega_2/4]$

$= I/4[\omega_1-\omega_2]^2$

21. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity 'ω' about the same axis. The final angular velocity of the combination of discs is :

a) $I_2\omega/(I_1 + I_2)$ b) ω **c) $I_1\omega/(I_1 + I_2)$** d) $(I_1 + I_2)\omega/I_1$

Solution : -

The initial angular momentum of the system is $I_1\omega$ and final angular momentum is $(I_1 + I_2)\omega'$

where

ω' = final angular velocity of combination of discs

On equating the initial and final angular

momentum, we get

$\omega' = I_1\omega/(I_1 + I_2)$

22. If a flywheel makes 120 rev/min, then its angular speed will be__

a) 8p rad/s b) 6 p rad/ s **c) 4 p rad / s** d) 2 p rad/s

Solution : -

Angular velocity of flywheel is given by

$\omega = 2\mathrm{p}v$

where, n is number of revolutions per second or frequency of revolution

Here, $n = 120\mathrm{rev/min}$

$\therefore \omega = \frac{2\pi\times120}{60} = 4\mathrm{prad/s}$

23. The moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its midpoint and perpendicular to its length is I_0. Its moment of inertia about an axis passing through one of its ends and perpendicular to its length is___

a) $I_0 + ML^{2/2}$ **b)** $I_0 + ML^2/4$ c) $I_0 + 2ML^2$ d) $I_0 + ML^2$

Solution : -

Applying theorem of parallel axis,

$I = I_{cm} + Md^2$

$I = I_0 + M(L/2)^2 = I_0 + ML^2/4$

24. A homogeneous disc of mass 2 kg and radius 15 cm is rotating about its axis (which is fixed) with an angular velocity of 4 radian/see with an angular velocity of 4 radian/sec. The linear momentum of the disc is:

a) 1.2 kg m/s b) 1.0 kg m/s c) 0.6 kg m/s d) none of above

Solution : -

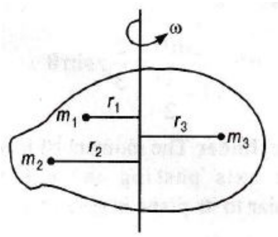
It is noted that the linear momentum of the disc is zero as it does not have translatory motion.

25. The angular momentum of a body with mass (m) moment of inertia (I) and angular velocity (w) rad/s is equal to :

a) 0 b) 60^2 c) $\frac{I}{0}$ **d)** $\frac{1}{\omega^2}$

Solution : -

Consider a rigid body rotating about a given axis with a uniform angular velocity ∞. Let the body consists of n particles of masses $m_1, m_2, m_3, \ldots, m_n$ at perpendicular distances $r_1, r_2, r_3, \ldots r_n$ respectively from the axis of rotation.



As the body is rigid, angular velocity w of all the particles is the same. However, as the distances of the particles from the axis of rotation are different, their linear velocities are different. If $v_1, v_2, v_3, \ldots, v_n$ are the linear velocities of the particles respectively, then

$v_1 = r_1\omega$

$v_2 = r_2\omega$

$v_3 = r_3\omega$

The linear momentum of this particle of mass m_1 is

$p_1 = m_1v_1 = m_1(r_1\omega)$

The angular momentum of this particle about the given axis

$= p_1 \times r_1 = (m_1r_1\omega) \times r_1$

$= m_1r_1^2\omega$

Similarly, angular momenta of other particles of the body about the given axis are

$m_2r_2^2\omega, m_3r_3^2\omega, \ldots m_nr_n^2\omega$

therefore Angular momentum of the body about the given axis.

$L = m_1r_3^2\omega + m_2r_2^2\omega + m_2r_3^2\omega + \ldots + m_nr_n^2\omega$

$= \left(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots + m_{11}r_{11}^2\right)\omega$

or $L = \left(\sum_{i=1}^{n} m_ir_i^2\right)\omega$

or $L = I\omega$

where, $I = \sum_{i=1}^{n} m_1r_i^2$ is moment of inertia of the body about the given axis.

26. Moment of the couple is called

a) angular momentum b) force **c) torque** d) impulse

Solution : -

Moment of the couple is called torque

27. A body is rotating with angular velocity $\vec{w} = (3\hat{i} - 4\hat{j} + \hat{k})$. The linear velocity of a point having position vector $\vec{r} = (5\hat{i} - 6\hat{j} + 6\hat{k})$ is

a) $6\hat{i} + 2\hat{j} - 3\hat{k}$ b) $18\hat{i} + 3\hat{j} - 2\hat{k}$ **c)** $-18\hat{i} - 13\hat{j} + 2\hat{k}$ d) $6\hat{i} - 2\hat{j} + 8\hat{k}$

Solution : -

$Here, \vec{w} = 3\hat{i} - 4\hat{j} + \hat{k}$

$\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$

As $\vec{v} = \vec{w} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$

$= \hat{i}(-24 - (-6)) + \hat{j}(5 - 18) + \hat{k}(-18 - (-20))$

$= -18\hat{i} - 13\hat{j} + 2\hat{k}$

28. A stone of mass 1 kg tied to a light inextensible string of length L = (10/3) m is whirling in a circular path of radius L in a vertical plane. If the ratio of the maximum tension in the string to the minimum tension in the string is 4 and if g is taken to be 10 m/s^2 the speed of the stone at the highest point of the circle is :

a) 20 m/s b) $10\sqrt{2}$ m/s c) $5\sqrt{2}$ m/s **d) 10 m/s**

Solution : -

Ratio of maximum tension (T_2) to minimum tension (T_1) is :

$T_2/T_1 = 4$

Also, $T_2 = 4T_1$

If the difference between two tension is same as 6 mg, so

$T_2 - T_1 = 6mg$

Further $4T_1 - T_1 = 6mg$

$3T_1 = 6mg$

$T_1 = 2mg$

It is noted that tension at top of circle is :

T_l + mg = mv_1^2/r

= mv_1^2/L

Now 2mg + mg = $mv_1^2/(10/3)$

v_1^2 = 3g × 10/3 = 10g

v_1 = $\sqrt{10g}$

= $\sqrt{10 \times 10}$

= 10 m/s

29. A thin uniform circular ring is rolling down an inclined plane of inclination 30° without slipping. Its linear acceleration along the inclined plane will be____

a) $\frac{g}{2}$ b) $\frac{g}{3}$ **c)** $\frac{g}{4}$ d) $\frac{2g}{3}$

Solution : -

Acceleration of the centre of mass of the rolling body is given by

$a = \frac{g \sin \theta}{1+\left(\frac{I}{MR^2}\right)}$

Moment of inertia of the ring about an axis perpendicular to the plane of the ring and passing through its centre is given by

$I = MR^2$

$\therefore a = \frac{g \sin \theta}{1+MR^2/MR^2}$

$= \frac{g \sin 30^\circ}{1+1} = \frac{g}{4}$

30. A ball of mass 0.25 kg attached to the end of a string of length 1.96 m is moving in a horizontal circle. The string will break if the tension is more than 25 N. What is the maximum speed with which the ball can be moved?

a) $14\ \mathrm{m/s}$ b) $3\ \mathrm{m/s}$ c) $3.92\ \mathrm{m/s}$ d) $5\ \mathrm{m/s}$

Solution : -

For a ball to move in horizontal circle, the ball should satisfy the condition

Tension in the string = Centripetal force

$\Rightarrow T_{\max}=\frac{Mv_{\max}^2}{R}$

$\Rightarrow v_{\max}=\sqrt{\frac{T_{\max}\cdot R}{M}}$

Making substitution, we obtain

$v_{\max}=\sqrt{\frac{25\times196}{0.25}}=\sqrt{196}=14\ \mathrm{m/s}$

Note

In a vertical circle, the tension at the highest point is zero and at lowest point is maximum.

31. In the question number 62, the linear acceleration of the rope is

a) 5 m s^{-2} **b) 10 m s^{-2}** c) 15 m s^{-2} d) 20 m s^{-2}

Solution : -

From above solution, α = 25 rad s^{-2}

Linear acceleration, a = αR = (25 rad s^{-2}) (40 x 10^{-2} m)

= 10 m s^{-2}

32. The speed of a homogenous solid sphere after rolling down an inclined plane of vertical height h from rest without sliding is :

a) $\sqrt{10gh/7}$ b) $\sqrt{gh}$ c) $\sqrt{6gh/5}$ d) $\sqrt{4gh/3}$

Solution : -

Total energy = Kinetic energy linear + Kinetic energy rolling

$E=\frac{1}{2}(mv^2)+\frac{1}{2}I\omega^2$

Here, ω = v/r

For solid sphere, moment of inertia, 1 = (2/5) mr^2

On substituting values of ω and 1

$E=\frac{1}{2}(mv^2)+(1/5)mv^2$

E = (7/10)mv^2

Now Potential energy = Total kinetic energy

mgh = (7/10)mv^2

∴ Velocity $v=\sqrt{\frac{10gh}{7}}$

33. Assertion: The moment of inertia of a rigid body reduces to its minimum value, when the axis of rotation passes through its centre of gravity.

Reason: The weight of a rigid body always acts through its centre of gravity.

a) If both assertion and reason are true and reason is the correct explanation of assertion.

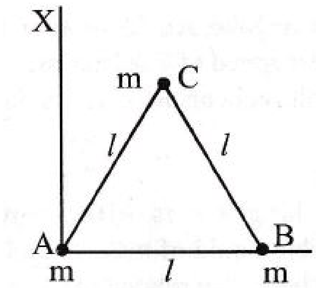
b) If both assertion and reason are true but reason is not the correct explanation of assertion.

c) If assertion is true but reason is false d) If both assertion and reason are false.

Solution : -

By theorem of parallel axis both statements are correct and reason is the correct explanation of assertion.

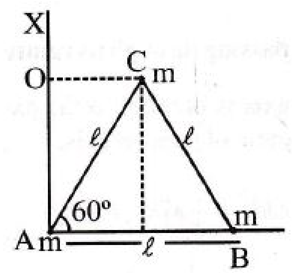
34. Three particles, each of mass m gram, are situated at the vertices of an equilateral triangle ABC of side/cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC,in gram cm^2 units will be ____________ .



a) 3/2 ml^2 b) 3/4 ml^2 c) 2 ml^2 **d) 5/4 ml^2**

Solution : -

$$I_{AX} = m(AB)^2 + m(OC)^2$$
$$= m\ell^2 + m(\ell\cos 60^\circ)^2$$
$$= m\ell^2 + m\ell^2/4 = 5/4 m\ell^2$$



35. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be____

a) $\frac{R^2}{K^2+R^2}$ b) $\frac{K^2+R^2}{R^2}$ c) $\frac{K^2}{R^2}$ **d)** $\frac{K^2}{K^2+R^2}$

Solution : -

Rotational energy $= \frac{1}{2}I(\omega)^2$

$= \frac{1}{2}\left(mK^2\right)\omega^2$

Linear kinetic energy $= \frac{1}{2}m\omega^2R^2$

therefore Required fraction

$$= \frac{\frac{1}{2}(mK^2)\omega^2}{\frac{1}{2}(mK^2)\omega^2+\frac{1}{2}m\omega^2R^2} = \frac{K^2}{K^2+R^2}$$

36. In the question number 66, if wheel starts from rest, what is the kinetic energy of the wheel when 2 m of the cord is unwound?

a) 20 J b) 25 J c) 45 J **d) 50 J**

Solution : -

Here, R = 20 cm = 0.2 m, M = 20 kg

As τ = FR = (25 N) (0.2 m) = 5 N m

Moment of inertia of flywheel about its axis is

$$I = \frac{MR^2}{2} = \frac{(20Kg)(0.2m)^2}{2} = 0.4kgm^2$$
$$As \quad \tau = I\alpha$$

$\therefore$ Angular acceleration of the wheel,

$$\alpha = \frac{\tau}{I} = \frac{5Nm}{0.4Kg \quad m^2} = 12.5rad \quad s^{-2}$$

Angular displacement of wheel,

$$\theta = \frac{Length\ of\ unwound\ string}{Radius\ of\ the\ world} = \frac{2m}{0.2m} = 10rad$$

Let Wbe final angular velocity.

As w^2 = w$_o^2$ + 2$\alpha\theta$

Since the wheel starts from rest, therefore $w_0 = 0$

$\therefore w^2 = 2 \times (12.5 \text{ rad s}^{-2})(10 \text{ rad}) = 250 \text{ rad}^2 \text{ s}^{-2}$

$\therefore$ Kinetic energy gained, $K = \frac{1}{2} I w^2$

$K = \frac{1}{2} \times 0.4 kg \quad m^2 \times 250 rad^2 s^{-1} = 50 \quad J$

37. A grindstone has a moment of inertia of 6 kg m^2. A constant torque is applied and the grindstone is found to have a speed of 150 rpm, 10 seconds after starting from rest. The torque is

a) $3\pi Nm$ b) 3 Nm c) $\frac{\pi}{3} Nm$ d) $4\pi Nm$

Solution : -

Here, I=6 Kg m^2, t=10 s, w_0=0

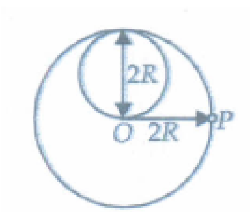
$v = 150 rpm = \frac{150}{60} rps = \frac{5}{2} rps$

$w = 2\pi v = 2\pi \times \frac{5}{2} = 5\pi rad \quad s^{-1}$

$\alpha = \frac{w - w_0}{t} = \frac{5\pi - 0}{10} = \frac{\pi}{2} rad \quad s^{-2}$

$\therefore \quad Torque, \tau = I\alpha = 6 \times \frac{\pi}{2} = 3\pi Nm$

38. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_p respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$



a) 13/37 **b) 37/13** c) 73/31 d) 8/13

Solution : -

Let M be mass of the whole disc.

Then, the mass of the removed disc $= \frac{M}{\pi(2R)^2} \pi R^2 = \frac{M}{4}$

So,moment of inertia of the remaining disc about an axis passing through O

$I_O = \frac{1}{2} M(2R)^2 - \left[\frac{1}{2}\left(\frac{M}{4}\right) R^2 + \frac{M}{4} R^2\right]$

$= 2MR^2 - \left[\frac{MR^2 + 2MR^2}{8}\right] = MR^2\left[2 + \frac{3}{8}\right] = \frac{13}{8} MR^2$

Moment of the inertia of the remaining disc about an axis passing through P is

$I_P = \left[\frac{1}{2} M(2R)^2 + M(2R)^2\right] - \left[\frac{1}{2}\left(\frac{M}{4}\right) R^2 + \frac{M}{4}(\sqrt{5}R)^2\right]$

$= \left[2MR^2 + 4MR^2\right] - \left[\frac{MR^2}{8} + \frac{5MR^2}{4}\right]$

$= 6MR^2 - \frac{11}{8} MR^2 = \frac{37}{8} MR^2$

$\therefore \frac{I_p}{I_O} = \frac{37}{4} \times \frac{8}{13} = \frac{37}{13}$

39. A disc is rotating with angular velocity ω. If a child sits on it, what is conserved?

a) Linear momentum **b) Angular momentum** c) Kinetic energy d) Moment of inertia

Solution : -

As the weight of child is acting downwards, torque is zero. When external torque is zero, angular momentum remains conserved.

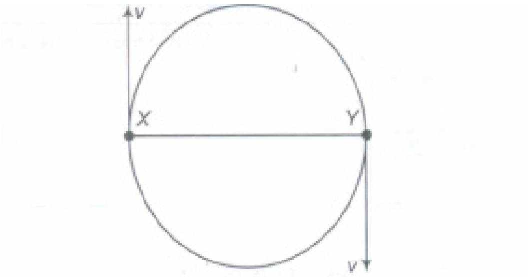
L= Iw = constant

40. Particle of mass m is moving in a horizontal circle of radius R with uniform speed v. When it moves from one point to a diametrically opposite point, its :

a) kinetic energy changes by $mv^2/4$ b) momentum does not change **c) momentum changes by 2 mv**
d) kinetic energy changes by mv^2

Solution : -

Now from the figure:



At point X, momentum = mv
At point Y, momentum = − mv
Hence, change in momentum
= mv − (−mv)
= 2mv
So, change in energy will be zero.

41. Assertion: A rigid body not fixed in some way can have either pure translation or a combination of translation and rotation.
Reason: In rotation about a fixed axis, every particle of the rigid body moves in a circle which lies in a plane perpendicular to the axis and has its centre on the axis.
a) If both assertion and reason are true and reason is the correct explanation of assertion
b) If both assertion and reason are true but reason is not the correct explanation of assertion.
c) If assertion is true but reason is false. d) If both assertion and reason are false.

Solution : -

A rigid body fixed at one point or along a line can have only rotational motion. A rigid body not fixed in some way can have either pure translation or a combination of translation and rotation.

42. A man is sitting with folded hands on a revolving table. Suddenly, he stretches his arms. Angular speed of the table would:
a) increase **b) decrease** c) remain the same d) nothing can be said

Solution : -

On stretching arms, distance K increases $I = MK^2$ increases. As $I\omega$ = constant, therefore, angular velocity ω decreases.

43. Consider a particle of mass m having linear momentum $\vec{p}$ at position $\vec{r}$ relative to the origin O. Let $\vec{L}$ be the angular momentum of the particle with respect to the origin. Which of the following equations correctly relate(s) $\vec{r}$, $\vec{p}$ and $\vec{L}$?
a) $\frac{d\vec{L}}{dt} + \vec{r} \times \frac{d\vec{p}}{dt} = 0$ b) $\frac{d\vec{L}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p} = 0$ c) $\frac{d\vec{L}}{dt} - \frac{d\vec{r}}{dt} \times \vec{p} = 0$ **d)** $\frac{d\vec{L}}{dt} - \vec{r} \times \frac{d\vec{p}}{dt} = 0$

Solution : -

As $\vec{L} = \vec{r} \times \vec{p}$
Differentiate both sides with respect to time, we get

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \vec{r} \times \frac{d\vec{p}}{dt} \qquad \left(\therefore \frac{d\vec{r}}{dt} \times \vec{p} = 0\right)$$

$$\frac{d\vec{L}}{dt} - \vec{r} \times \frac{d\vec{p}}{dt} = 0$$

44. A B C is a triangular plate of uniform thickness. The sides are in the ratio shown in the figure. I_{AB}, I_{BC} and I_{CA} are the moments of inertia of the plate about A B, B C and C A as axes respectively. Which one of the following relations is correct?

a) $I_{AB} > I_{BC}$ **b)** $I_{BC} > I_{AC}$ c) $I_{AB} + I_{BC} = I_{CA}$ d) I_{CA} is maximum

Solution : -

Moment of inertia of the triangular plate is maximum about the shortest side because effective distance of mass distribution about this side is maximum. Since, distances of centre of mass from the sides are related as

$X_{BC} < X_{AB} < X_{AC}$

Therefore $I_{BC} > I_{AB} > I_{AC}$ or $I_{BC} > I_{AC}$

45. Assertion: A boiled egg can be easily distinguished from a raw unboiled egg by spinning.

Reason : The hard boiled egg has a moment of inertia which is more than that of the raw egg.

a) If both assertion and reason are true and reason is the correct explanation of assertion.

b) If both assertion and reason are true but reason is not the correct explanation of assertion

c) If assertion is true but reason is false d) If both assertion and reason are false.

Solution : -

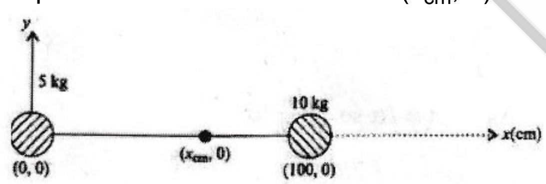
A hard boiled egg behaves as a rigid body whereas the raw egg has liquid in it which also moves. On spinning both the eggs with the same external torque, the hard boiled egg spins faster and the rotation of the raw egg is slower. This is due to the liquid in the raw egg which tends to move away from the axis of rotation thereby decreasing the effectiveness of the external torque.

46. Two particles of mass 5 kg and 10 kg respectively are attached to the two ends of a rigid rod of length 1 m with negligible mass. The centre of mass of the system from the 5 kg particle of nearly at a distance of____

a) 80 cm b) 33 cm c) 50 cm **d) 67 cm**

Solution : -

Let position of centre of mass be (x_{cm}, 0)



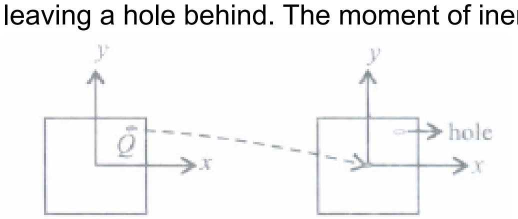
$x_{cm} = \frac{m_1x_1+m_2x_2}{m_1+m_2}$

$= \frac{5\times0+100\times10}{5+10}$

$= \frac{200}{2} = 66.66 \text{ cm}$

$x_{\text{cm}} = 67 \text{ cm}$

47. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind. The moment of inertia about the z-axis is then



a) increased **b) decreased** c) the same d) changed in unpredicted manner

Solution : -

According to the theorem of perpendicular axes

$I_z = I_x + I_y$

With the hole, I_x and I_y both decrease. Gluing the removed piece at the centre of square plate does not affect I_z.

Hence, I_z decrease overall.

48. Which of the following statements is correct?

a) For a general translational motion, momentum $\vec{p}$ and velocity $\vec{v}$ need not be parallel.

b) For a general rotational motion, angular momentum $\vec{L}$. and angular velocity $\vec{w}$ are always parallel.

c) For a general translational motion, acceleration $\vec{a}$ and velocity $\vec{v}$ are always parallel.

d) For a general rotational motion, angular momentum $\vec{L}$ and angular velocity $\vec{w}$ need not be parallel.

Solution : -

For a general rotational motion, angular momentum $\vec{L}$ and angular velocity $\vec{w}$ need not be parallel. For a general translation motion $\vec{p}$ and $\vec{v}$ are always parallel.

49. A solid homogeneous sphere of mass M and radius R is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of the sphere___

a) total kinetic energy is conserved

b) the angular momentum of the sphere about the point of contact with the plane is conserved

c) only the rotational kinetic energy about the centre of mass is conserved

d) angular momentum about the centre of mass is conserved

Solution : -

Angular momentum about the point of contact, for solid homogeneous sphere of mass M and radius R is conserved.

50. A rod of we ight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from l. The normal reaction on A is_____

a) $\frac{Wd}{x}$ b) $\frac{W(d-x)}{x}$ **c)** $\frac{W(d-x)}{d}$ d) $\frac{Wx}{d}$

Solution : -

Balancing torque about B, we have

$N_A(d) = W(d-x)$

$N_A = \frac{W(d-x)}{d}$